9.88

$m_A = 7\text{ kg}, \; m_B = 9\text{ kg}, \; v_A^- = 6\text{ m/s}, \; v_B^- = 2\text{ m/s}, \; e = 0.5$

$$m_A v_A^- + m_B v_B^- = m_A v_A^+ + m_B v_B^+ \quad (1)$$

$$v_B^+ - v_A^+ = e\,(v_A^- - v_B^-)$$

In matrix form,

$$\begin{bmatrix} m_A & m_B \\ -1 & 1 \end{bmatrix} \begin{pmatrix} v_A^+ \\ v_B^+ \end{pmatrix} = \begin{bmatrix} m_A & m_B \\ e & -e \end{bmatrix} \begin{pmatrix} v_A^- \\ v_B^- \end{pmatrix}$$

$$\begin{bmatrix} 7 & 9 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} v_A^+ \\ v_B^+ \end{pmatrix} = \begin{bmatrix} 7 & 9 \\ 0.5 & -0.5 \end{bmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 60 \\ 2 \end{pmatrix}$$

Solving, $\left[\begin{array}{cc|c} 7 & 9 & 60 \\ -1 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 9/7 & 60/7 \\ -1 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 9/7 & 60/7 \\ 0 & 16/7 & 74/7 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & 21/8 \\ 0 & 16/7 & 74/7 \end{array}\right] \quad \therefore \quad \boxed{\begin{array}{l} v_A^+ = 2.625\text{ m/s} \\ v_B^+ = 4.625\text{ m/s} \end{array}}$

9.89

$m_A = 2\text{ kg}, m_B = 5\text{ kg}, v_A^- = 5\text{ m/s}, v_B^- = 1\text{ m/s}$

(1) $m_A v_A^- + m_B v_B^- = m_A v_A^+ + m_B v_B^+$

(2) $v_B^+ - v_A^+ = e(v_A^- - v_B^-)$, $e = 0.5$

From (2), $v_B^+ = v_A^+ + 0.5(5-1) = v_A^+ + 2\text{ m/s}$

Plug into (1), $2(5) + 5(1) = 2v_A^+ + 5(v_A^+ + 2)$

OR $15\text{ kg m/s} = 7v_A^+ + 10\text{ m/s}$

$\therefore v_A^+ = 0.714\text{ m/s}$

$v_B^+ = 2.714\text{ m/s}$

$E_k^- = \frac{1}{2} m_A v_A^{-2} + \frac{1}{2} m_B v_B^{-2} = \frac{1}{2}(2\text{ kg})(5\text{ m/s})^2 + \frac{1}{2}(5\text{ kg})(1\text{ m/s})^2$
$= 27.5\text{ J}$

$E_k^+ = \frac{1}{2} m_A v_A^{+2} + \frac{1}{2} m_B v_B^{+2} = \frac{1}{2}(2\text{ kg})(.714\text{ m/s})^2 + \frac{1}{2}(5\text{ kg})(2.714\text{ m/s})^2$
$= 18.93\text{ J}$

$\therefore \Delta E_k = 27.5 - 18.93 = \boxed{8.57\text{ J lost}}$

9.92

Basketball with mass m_b dropped from height h, $e = e_b$

Small rubber ball with mass m_r, $e = e_r$

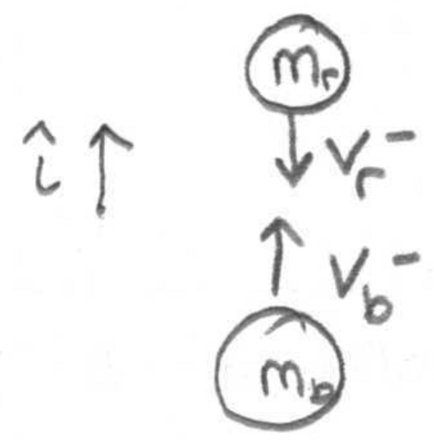
a) Treat this as two collisions:

i) basketball hits ground

$V_0 = 0$, cons. of energy $V_f = -\sqrt{2gh}$ (before hit)

After collison $V = e_b V_f = e_b\sqrt{2gh}$

ii) basketball and rubber ball collide



$$V_r^- = -\sqrt{2gh}, \quad V_b^- = e_b\sqrt{2gh}$$

$$m_b V_b^- + m_r V_r^- = m_b V_b^+ + m_r V_r^+ \quad (1)$$

$$V_r^+ - V_b^+ = e_r(V_b^- - V_r^-) \quad (2)$$

From (2), $V_b^+ = V_r^+ - e_r V_b^- + e_r V_r^-$

$$= V_r^+ - e_r e_b\sqrt{2gh} - e_r\sqrt{2gh}$$

$$= V_r^+ - e_r\sqrt{2gh}(1+e_b)$$

Plug into (1), $m_b e_b\sqrt{2gh} - m_r\sqrt{2gh} = m_r V_r^+ + m_b\left(V_r^+ - e_r\sqrt{2gh}(1+e_b)\right)$

OR $\sqrt{2gh}(m_b e_b - m_r) = m_r V_r^+ + m_b V_r^+ - m_b e_r\sqrt{2gh}(1+e_b)$

$$= V_r^+(m_r + m_b) - m_b e_r\sqrt{2gh}(1+e_b)$$

$$\therefore V_r^+(m_r + m_b) = \sqrt{2gh}\left[m_b e_b - m_r + m_b e_r(1+e_b)\right]$$

OR $$V_r^+ = \frac{\sqrt{2gh}\left[m_b e_b - m_r + m_b e_r(1+e_b)\right]}{m_r + m_b}$$

next

Conservation of energy:

$$\cancel{m_r} g h_r = \frac{1}{2} \cancel{m_r} (v_r^+)^2 \quad \therefore \quad h_r = \frac{1}{2g}(v_r^+)^2$$

$$h_r = \frac{1}{\cancel{2g}}(\cancel{2g}h)\left[\frac{m_b e_b - m_r + m_b e_r (1+e_b)}{m_r + m_b}\right]^2$$

$$\therefore \quad \boxed{h_r = h\left[\frac{m_b e_b - m_r + m_b e_r (1+e_b)}{m_b + m_r}\right]^2}$$

b) To maximize h_r, we can begin by recognizing that letting $e_b = e_r = 1$ maximizes the numerator of the bracketed expression.

$$\therefore \quad h_r = h\left(\frac{m_b - m_r + 2m_b}{m_b + m_r}\right)^2 = h\left(\frac{3m_b - m_r}{m_b + m_r}\right)^2$$

This is maximized by increasing m_b and decreasing m_r.

$\therefore$ We want $e_b = e_r = 1$ and the largest ratio of m_b to m_r possible.

$$h_{r,max} = h\left(\frac{3m_b - 0}{m_b + 0}\right)^2 = 9h$$

$$\boxed{\text{Theoretical max } h_r = 9h}$$

9.93

Given: m_A, m_B, V_B, V_A

$$\Delta E_k = \tfrac{1}{2} m_A V_A^2 + \tfrac{1}{2} m_B V_B^2 - \tfrac{1}{2} m_A V_{Af}^2 - \tfrac{1}{2} m_B V_{Bf}^2$$

$$2\Delta E_k = m_A (V_A^2 - V_{Af}^2) + m_B (V_B^2 - V_{Bf}^2)$$

$$2\Delta E_k = m_A (V_A - V_{Af})(V_A + V_{Af}) + m_B (V_B - V_{Bf})(V_B + V_{Bf})$$

$$= (m_A V_A - m_A V_{Af})(V_A + V_{Af}) + (m_B V_B - m_B V_{Bf})(V_B + V_{Bf})$$

From conservation of momentum:

$$m_A V_A + m_B V_B = m_A V_{Af} + m_B V_{Bf}$$

OR $-(m_A V_A - m_A V_{Af}) = m_B V_B - m_B V_{Bf}$

$$\therefore\ 2\Delta E_k = (m_A V_A - m_A V_{Af})(V_A + V_{Af}) - (m_A V_A - m_A V_{Af})(V_B + V_{Bf})$$

$$= (m_A V_A - m_A V_{Af})(V_A + V_{Af} - V_B - V_{Bf})$$

$$= m_A (V_A - V_{Af})(V_A + V_{Af} - V_B - V_{Bf})$$

Newton's Constitutive Law: $e = \dfrac{V_{Bf} - V_{Af}}{V_A - V_B}$

OR $e(V_A - V_B) = V_{Bf} - V_{Af}$

$e(V_B - V_A) = V_{Af} - V_{Bf}$

$$\therefore\ 2\Delta E_k = m_A (V_A - V_{Af})(V_A + eV_B - eV_A - V_B)$$

$$= m_A (V_A - V_{Af})\left[V_A(1-e) - V_B(1-e)\right]$$

$$= m_A (V_A - V_{Af})(1-e)(V_A - V_B)$$

We know $V_A > V_B$, $V_A > V_{Af}$, so the sign of ΔE_k is the same as the sign of $1-e$. $\Delta E_k = E_{ko} - E_{kf}$, which must be > 0. If $e > 1$, $\Delta E_k < 0$ and we have an increase in kinetic energy. So $|e| \le 1$.